An Approach to Noisy Image Skeletonization using Morphological Methods

Dr M Ashok , J SreeDevi and Dr M Rama Bai

Abstract— This paper presents an approach for removing noise and extracting the skeleton of the given image. Noise should be removed while keeping the fine detail of the image intact. A noise removal algorithm which can remove noise while retaining fine image is presented in this paper. The algorithm uses the morphological operators for removal of noise and then extracts the skeleton of the image and reconstructs the fine image. Objective distortion measurements including PSNR and MSE show that our algorithm gives better quality images compared with other methods.

Index Terms— *Mathematical morphology, Salt and pepper noise, skeleton, reconstruction..*

1 INTRODUCTION

Noise may appear in images during data acquisition such as scanning [1],[2]. The noise should be removed prior to performing image analysis processes. The difficulty in removing salt and pepper noise from binary image is due to the fact that image data as well as the noise share the same small set of values (either 0 or 1) which complicate the process of detecting and removing the noise. This is different from gray images where salt/pepper noise could be distinguished as pixels having big difference in gray level values compared with their neighborhood. Many algorithms have been developed to remove salt and pepper noise in document images with different performance in removing noise and retaining fine details of the image. Most of the methods easily remove isolated pixels while leaving some noise. In this paper many algorithms will be studied. Then a procedure that enhances removal of salt and pepper noise is proposed. This procedure is then incorporated with a third party method to produce an algorithm that removes noise and retains image.

Mathematical morphology is a well-founded non-linear theory of image processing [2], [3]. Its geometry-oriented nature provides an efficient framework to analyze object shape characteristics such as size and connectivity, which are not easily accessed by linear approaches. Morphological operations take into consideration the geometrical shape of the image objects to be analyzed. The initial form of mathematical morphology is applied to binary images and usually referred to as standard mathematical morphology in the literature in order to be discriminated by its later extensions such as the gray scale and the soft mathematical morphology. Mathematical morphology is theoretically founded on set theory. It contributes a wide range of operators to image processing, based on a few simple mathematical concepts. The operators are particularly useful for the analysis of binary images, boundary detection, noise removal, image enhancement and image segmentation. The advantages of morphological approaches over linear approaches are direct geometric interpretation, simplicity and efficiency in hardware implementation.

The hardware complexity of implementing morphological operations depends on the size of the structuring elements. The complexity increases even exponentially in some cases. The known hardware implementations of morphological operations are capable of processing structuring elements only up to 3 × 3 pixels. If higher order structuring elements are needed, they are decomposed into smaller elements. One decomposition strategy is, for example, to present the structuring element as successive dilation of smaller structuring elements. This is known as the “chain rule for dilation”. But all structuring elements cannot be decomposed.

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Morphological Processing and Transforms

An image is a function of two, real (coordinate) variables *a* (x, y) or two, discrete variables *a* [m, n]. An alternative definition of an image can be based on the notion that an image consists of a set (or collection) of either continuous or discrete coordinates. In a sense the set corresponds to the points or pixels that belong to the objects in the image. For the moment the present paper will consider the pixel values to be binary.

The basic *Minkowski set operations* addition and subtraction can be defined, based on assumptions that the individual elements that comprise B are not only pixels but also vectors as they have a clear coordinate position with respect to [0,0]. For given two sets A and B, the operations are computed by the following equations 1 and 2.

Minkowski addition – $$A \oplus B = \bigcup_{\beta \in B} (A + \beta) \quad (1)$$

Minkowski subtraction – $$A \Theta B = \bigcap_{\beta \in B} (A + \beta) \quad (2)$$

Dilation and Erosion

The fundamental mathematical morphology operations *dilation* and *erosion* based on Minkowski algebra are defined by the following equations 3 and 4.

Dilation- $$D(A,B) = A \oplus B = \bigcup_{\beta \in B} (A + \beta) \quad (3)$$

Erosion- $$E(A,B) = A \Theta (-B) = \bigcup_{\beta \in B} (A - \beta) \quad (4)$$

Dilation, in general, causes objects to dilate or grow in size; *erosion* causes objects to shrink. The amount and the way they grow or shrink depend upon the choice of the structuring element. Dilating or eroding without specifying the structural element makes no more sense, than trying to low pass filter an image without specifying the filter. The two most common structuring elements (given a Cartesian grid) are the 4-connected and 8-connected sets, N_4 and N_8.

Dilation and *Erosion* have the following properties computed by the following equations 5-9.

Commutative– $$D(A,B) = A \oplus B = B \oplus A = D(B,A) \quad (5)$$

Non-Commutative – $$E(A,B) \neq E(B,A) \quad (6)$$

Associative– $$A \oplus (B \oplus C) = (A \oplus B) \oplus C \quad (7)$$

Translation Invariance– $$A \oplus (B + x) = (A \oplus B) + x \quad (8)$$

Duality – $$\begin{aligned} D^c(A,B) &= E(A^c, -B) \\ E^c(A,B) &= D(A^c, -B) \end{aligned} \quad (9)$$

with A as an object and A^c as the background, equation says that the *dilation* of an object is equivalent to the *erosion* of the background. Likewise, the *erosion* of the object is equivalent to the *dilation* of the background except for the special cases given in the following equation 10.

Non-Inverses – $$D(E(A,B),B) \neq A \neq E(D(A,B),B) \quad (10)$$

Erosion has the following translation property as given by the following equation 11.

Translation Invariance–

$$A \Theta (B + x) = (A + x) \Theta B = (A \Theta B) + x \quad (11)$$

Opening and Closing

One can combine *dilation* with *erosion* to build two important higher order operations called opening and closing given by the following equations 12 and 13.

Opening– $$O(A,B) = A \circ B = D(E(A,B),B) \quad (12)$$

Closing – $$C(A,B) = A \bullet B = E(D(A,-B),-B) \quad (13)$$

The *opening* and *closing* have the following properties computed by the equations 14 - 21.

Duality – $$\begin{aligned} C^c(A,B) &= O(A^c, B) \\ O^c(A,B) &= C(A^c, B) \end{aligned} \quad (14)$$

Translation – $$\begin{aligned} O(A + x, B) &= O(A,B) + x \\ C(A + x, B) &= C(A,B) + x \end{aligned} \quad (15)$$

For the *opening* with structuring element B and images A, A_1, and A_2, where A_1 is a sub image of $A_2 (A_1 \subseteq A)$:

Antiextensivity – $$O(A,B) \subseteq A \quad (16)$$

Increasing monotonicity – $$O(A_1, B) \subseteq O(A_2, B) \quad (17)$$

Idempotence – $O(O(A,B),B) = O(A,B)$ (18)

For the *closing* with structuring element *B* and images *A*, A_1, and A_2, where A_1 is a sub image of A_2 ($A_1 \subseteq A_2$):

Extensivity – $A \subseteq C(A,B)$ (19)

Increasing monotonicity – $C(A_1,B) \subseteq C(A_2,B)$ (20)

Idempotence – $C(C(A,B),B) = C(A,B)$ (21)

The two properties given by equations 20 and 21 are so important to mathematical morphology that they can be considered as the reason for defining *erosion* with -*B* instead of *B* in equation 13.

Skeleton

The informal definition of a skeleton is a line representation of an object that is: *i*) one-pixel thick *ii*) through the middle of the object and iii) preserves the topology of the object. These are not always realizable. Fig 1 shows counter examples for skeleton definition.

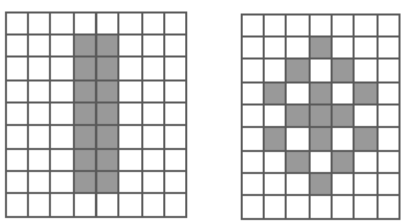


Fig. 1 (a) (b) Counter examples to the Three Requirements.

In the first example, Fig.1 (a), it is not possible to generate a line that is one pixel thick and in the center of an object while generating a path that reflects the simplicity of the object. In Fig. 1 (b) it is not possible to remove a pixel from the 8-connected object and simultaneously preserve the topology, the notion of connectedness of the object. Nevertheless, there are a variety of techniques that attempt to achieve this goal and to produce a *skeleton*.

A basic formulation is based on the work of Lantuejoul. The *skeleton subset* $S_k(A)$ is defined by the following equation 23.

Skeleton subsets– $S_k(A) = E(A,kB) - [E(A,kB) \circ B]$ (23)

$k = 0,1,..K$

where *K* is the largest value of *k* before the set $S_k(A)$ becomes empty. The structuring element *B* is chosen to approximate a circular disc, that is, convex, bounded and symmetric. The *skeleton* is then the union of the skeleton subsets as given by the equation 24.

Skeleton – $S(A) = \bigcup_{k=0}^{K} S_k(A)$ (24)

The original object can be reconstructed with the given knowledge of the skeleton subsets $S_k(A)$, the structuring element *B*, and *k* as given by the following equation 25.

Reconstruction – $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$ (25)

The skeleton computed by the equation 24 preserve all the above properties.

Basic Skeletonization Techniques

In this section skeletonization techniques using mathematical morphology, is discussed.

Medial Axis Transform (MAT)

MAT [4]- [8] is unique. The original image can be reconstructed from its medial axis transform. The MAT evolved through Blum's work on animal vision systems. His interest involved how animal vision systems extract geometric shape information. There exists a wide range application that finds a minimal representation of an image useful. The MAT is especially useful for image compression since reconstruction of an image from its MAT is possible.

Let $X \subset R^2, a \in \{0,1\}^x$, and let *A* denotes the support of *a*. The MA is the set, $M \subseteq A$, consisting of those points *x* for which there exists a ball of radius r_x, centered at *x*, that is contained in *A* and intersects the boundary of *A* in at least two distinct points. The MAT *m* is a gray level image defined over *x* by the following equation 26.

$$m(x) = \begin{cases} r_x \; if & x \in M \\ 0 & otherwise \end{cases} \tag{26}$$

The reconstruction of the domain of the original image *a* in terms of the MA *m* is given by the following equation 27.

$$A = \bigcup_{x \in M} B_{m(x)}(x) \tag{27}$$

The original image *a* is computed by the following equation 28.

$$a(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \in X \setminus A \end{cases} \tag{28}$$

Let *a* denote the source image. Usually, the neighbourhood *N* is a digital disk of a specified radius. The shape of the digital disk depends on the distance function used. The MAT will be stored in the image variable *m*. Reconstruction from MAT is given by the following equation 29.

$$a = \bigcup_{k=1}^{i} (x_{=k}(m) \oplus N^{k-1}) \tag{29}$$

Because each MAT value is greater than the associated neighborhood radius, the transformation encodes isolated points (with value 1) and yields exact reconstruction.

Since the present is mainly motivated by the concepts of mathematical morphology, an in depth study on the basic concepts was done, and skeletonization algorithms using morphology is discussed in it.

2. METHODOLOGY FOR PROPOSED WORK

An algorithm of the proposed method is given in the form of a flow chart and is shown in Fig 2.

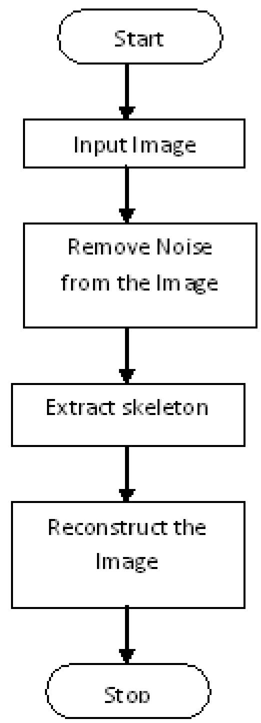


Fig.2: Flow chart of the proposed method

In this proposed paper the study removes salt and pepper noise from the image by using morphological operators.

Fig.3: Original Image Fig. 4: Noisy image

In the second step the noise is removed by using the basic morphological operators (dilation and erosion). The image obtained after removing the noise is as below Fig. 5.

Fig.5: Noise filtered image

Next the skeleton is extracted from the above noise free image. The skeleton of the image is also obtained by using the basic morphological operators. The skeleton got is as shown below in Fig. 6.

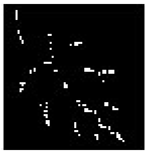


Fig.6: Skeleton obtained by using morphological operators.

To the skeletonized image reconstruction techniques is applied to extract the original. The reconstructed image is as shown below in Fig. 7.

Fig. 7: Reconstructed Image

3. RESULTS and ANALYSIS

The proposed algorithm for removal of noise is compared with order statistic and average filters. The Peak Signal to Noise Ratio (PSNR) is more for the proposed algorithm than the other algorithms and Mean Square Error (MSE) is less for the proposed algorithm than the other algorithms as shown in Table1.

Image	Proposed Method		Average filter		Order statistic filter	
	PSNR	MSE	PSNR	MSE	PSNR	MSE
alphabets	29.888	7.469	27.251	11.066	16.67	37.387
child	31.155	7.059	29.915	8.142	20.822	23.195
dog	19.461	27.13	15.816	41.279	6.576	119.601
dolphin	32.758	5.869	30.257	7.828	21.968	20.353
fish	16.828	36.74	14.453	48.293	6.586	119.462
leaf	18.618	29.9	17.211	35.153	7.577	106.583
numbers	30.664	7.469	27.036	11.342	16.642	37.521
rabbit	23.5	17.04	17.173	35.306	8.985	90.631
tree	33.44	5.426	32.201	6.258	25.277	13.888

Table 1: Comparison of PSNR and MSE values for different images with different filters.

The error functions reflect the fact between the original image and the reconstructed image. Table 1 shows the error rates of reconstructed images with original image using proposed method, order statistic filter and average filter algorithm respectively.

The error rate is evaluated on nine different images. It is evident that the error rate of the present method is reduced when compared with other two algorithms. One more interesting point is that the error rate of the proposed method is less than the other two methods for all images by using error functions. The PSNR is high for the proposed method for all images. It indicates that it has high signal to noise ratio. The MSE is low for proposed method for all images, compared with other two algorithms.

The graphical representation of the comparison of PSNR values of existing methods with the proposed method which is given in Table 1 is shown in Fig.8 and Fig 9. From the graph it is clear that the proposed method for noise removal, skeletonization and reconstruction of image give good Peak Signal-to-Noise Ratio (PSNR) and less Mean Square Error (MSE) than the existing algorithms.

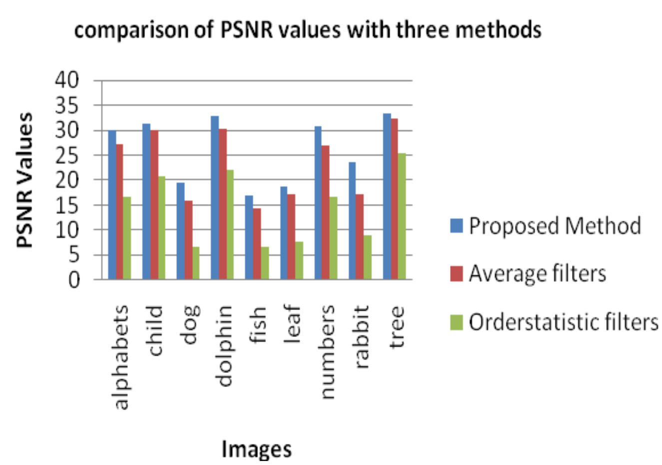


Fig.8: Comparison graph of PSNR values of existing algo-

rithms with proposed method.

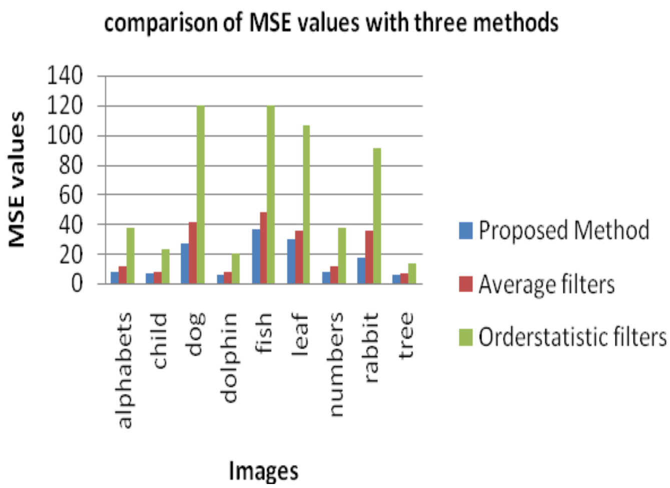
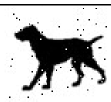
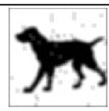
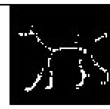
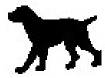


Fig.9: Comparison graph of MSE values of existing algorithms with proposed method.

a. EXPERIMENTAL RESULTS FOR DIFFERENT IMAGES

original	Noisy	Average filter	Skeleton	Reconstruction
				
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			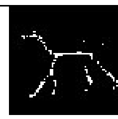	
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			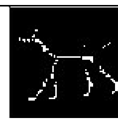	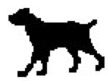
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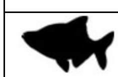		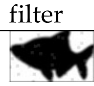		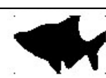
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		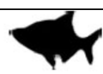		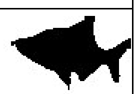
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		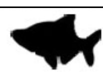		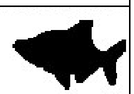
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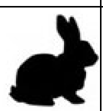		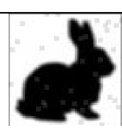	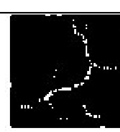	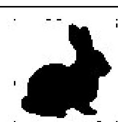
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			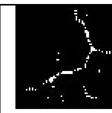	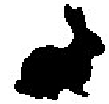
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		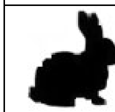	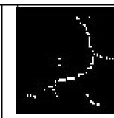	

4. CONCLUSION

In this paper, we propose a method for removal of salt and pepper noise, skeletonization and reconstruction of image using morphological methods. The proposed method gives better performance in comparison with Average Filter and Order statistic filter. The experimental results using error functions on different images show that the proposed method produces more clarity-based representation than the other two approaches.

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